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Practical asymptotics

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Abstract. The term *practical asymptotics* is explained. It is argued that many practical problems are amenable to and will benefit from the asymptotic approach. Fifteen papers by authors who are active in a wide range of disciplines demonstrate this. It is argued that the teaching of asymptotic methods should remain an integral part of any sophisticated numerics curriculum, raising it far above the level of mere number crunching.

Key words: asymptotics, asymptotics teaching, non-dimensionalisation, numerics, real-world problems.

A few years ago this journal published a special issue entitled *Large-Scale Numerical Modelling of Problems Involving the Navier-Stokes Equations* (32 (1997) 101–280). That appeared to be a very timely publication. Indeed, these days applied mathematics seems, at first sight anyway, to become more and more dominated by direct numerical simulation. Admittedly, this leads to new insights which, it would seem, could not have been attained by other means. The purpose of that issue was to highlight that particular aspect.

The interest of large sections of the community of applied mathematicians being focussed in the above direction nowadays, the older and broad subject of asymptotics seems to be losing the popularity it once enjoyed and the impact it had. This is partly because many believe that asymptotics deals with exceptional cases which are usually outside the practical domain. However, this is a misconception! To make this clear, we offer the following argument:

Having non-dimensionalised your problem, consider a typical dimensionless group

$$N_D = \alpha_1^{k_1}\alpha_2^{k_2}\alpha_3^{k_3}\ldots,$$

where the k_i are real-valued exponents that can be both positive and negative. We know that, within a given set of units, each of the physical parameters α_i can assume values that may vary widely depending upon the problem in hand, *i.e.* $\alpha_i \sim 10^{n_i}$, where the integers n_i can range from large negative to large positive values. Thus,

$$N_D \sim 10^p\,, \qquad \left(p = \sum_i n_i k_i\right)$$

and we can ask ourselves: what is the probability that $p \sim 0$? Of course, p is most likely to be less than -1 or larger than 1, which means that the problem can be simplified through the application of asymptotic techniques, *without seriously affecting the practical usefulness of the reduced model*. The technique, or rather the craft, by which the reduction is accomplished is called *Practical Asymptotics.* It is a highly intuitive process and is based, to a large extent, on physical reasoning.

Papers which demonstrate the power of practical asymptotics should involve both large-scale computations on the full model and computations based on the asymptotically reduced model. A comparison between the two should then illustrate a considerable reduction in computing effort in terms of cpu time. Further emphasis should be put on the explicit representation of trends, physical relationships, rules, laws, etc., as expressed by mathematical formulae resulting from the asymptotics with numerically generated coefficients. This, then, should contrast with the practice of large-scale numerics which usually results in the presentation of numbers, (colour) plates or contour plots relating to specific examples.

In the present issue fifteen papers have been collected, each seeking to demonstrate the usefulness and practical nature of the asymptotic approach for the discipline from which it arose. These papers address subjects as varied as: the production of glass bottles, semiconductors, surface-tension-driven flows, microwave-assisted joining of tubes, viscous-inviscid interaction, industrial limit cycles, heat exchangers used in the production of foodstuffs, water waves generated by bottom topography, channel flows, chemical-clock reactions, Stokes flows, separated flows, superconductivity, flows over fibrous plates, multiscale problems arising in low-Mach-number flows; a wide range of subjects indeed!

We hope that this issue will demonstrate the continuing usefulness and validity of the asymptotic approach in reducing the complexity of mathematical models to a practical minimum, *i.e.*, without unduly sacrificing their accuracy, thus achieving large reductions in computing effort and increased physical understanding. We believe that the teaching of asymptotic methods should remain an integral part of the academic curriculum as an indispensable tool for the numericist wishing to tackle problems of ever-increasing complexity.